

Vectors

Some Basic Concepts

Scalars

ONLY Magnitude

No direction.

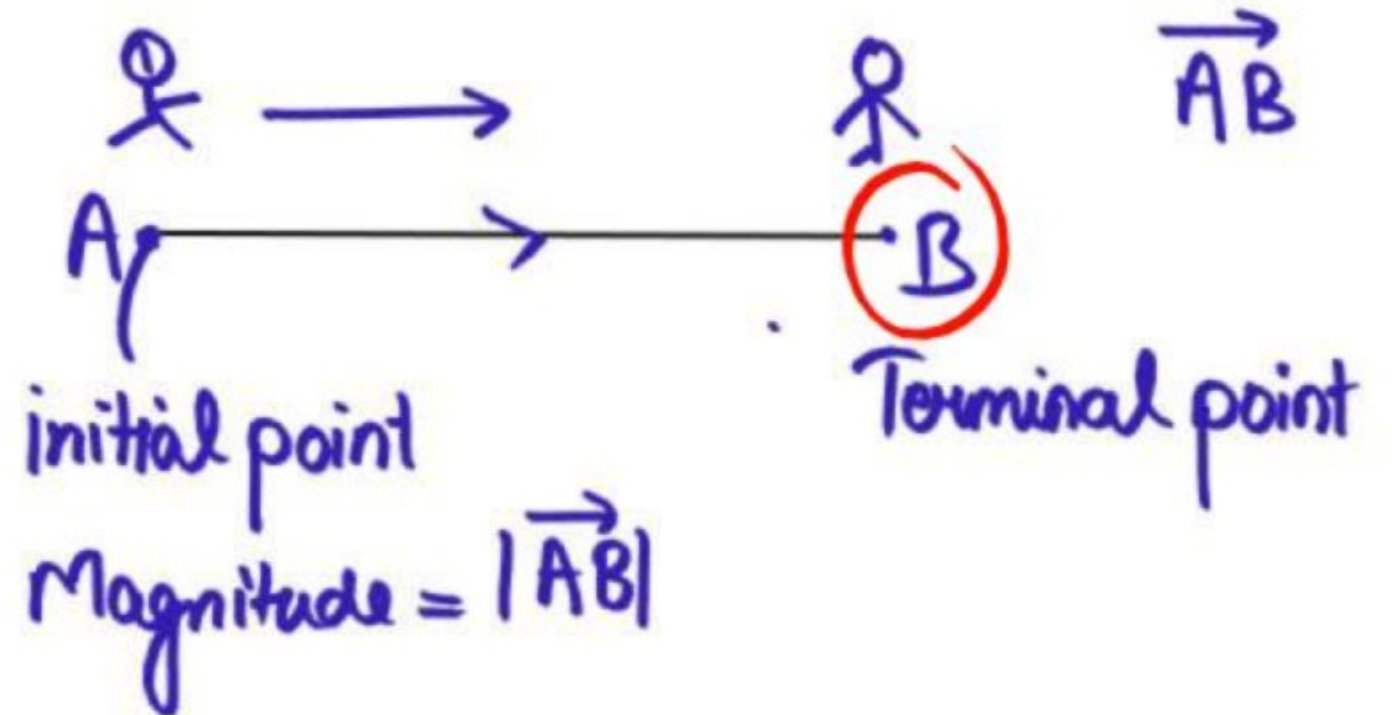


$\hookrightarrow$ distance, Mass, temp, time

Vectors

Magnitude + Direction

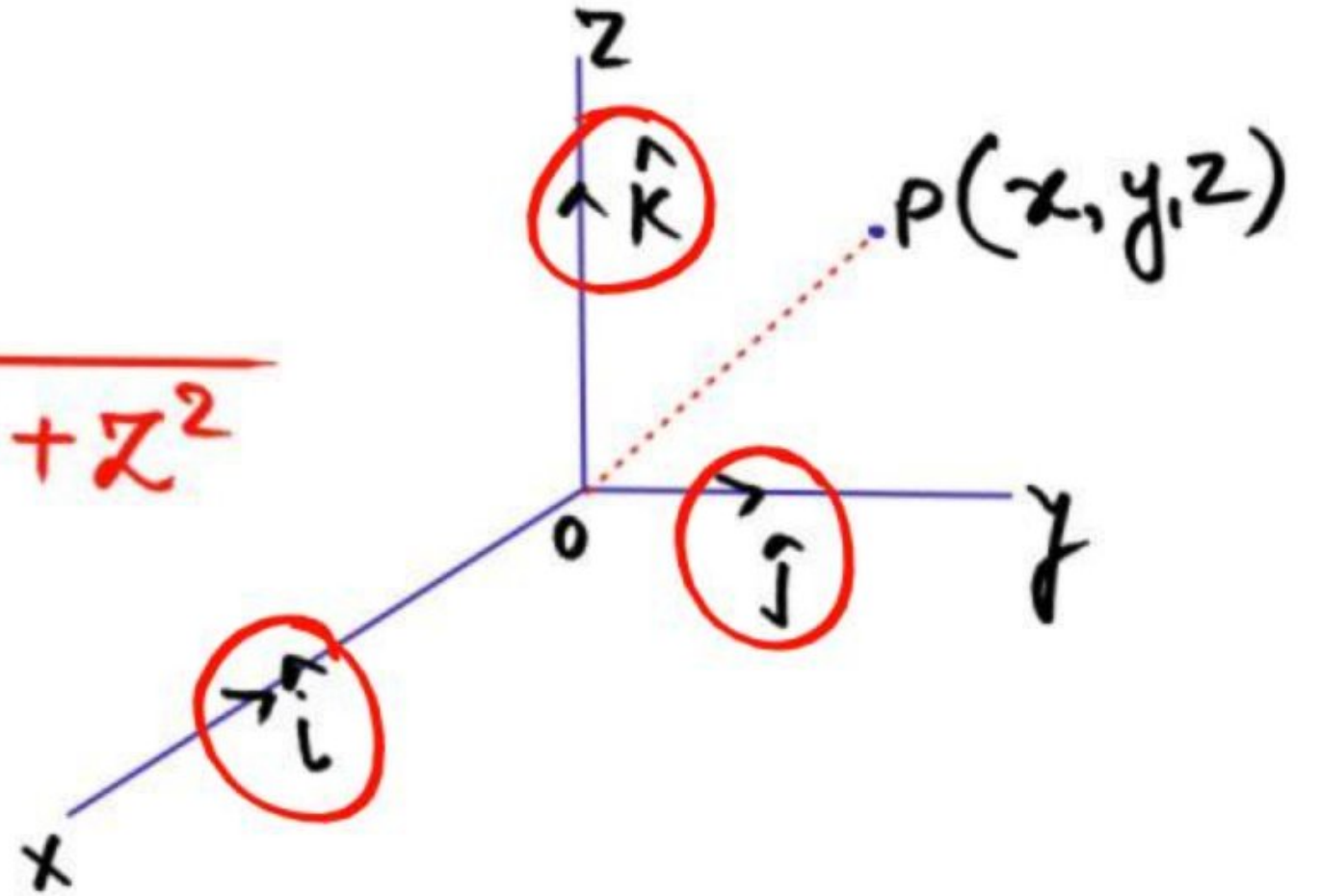
$\hookrightarrow$ displacement, force, weight



Position vector of a point

$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

Magnitude of $\vec{OP} = |\vec{OP}| = \sqrt{x^2 + y^2 + z^2}$



Types of Vectors

Coinitial Vector

Same initial point

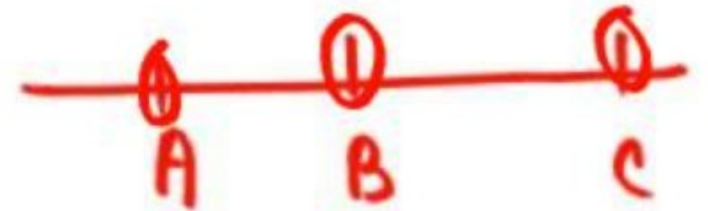


Coplanar Vector

Same plane

Collinear Vector

Vectors $\parallel$ to the same line



Free Vector

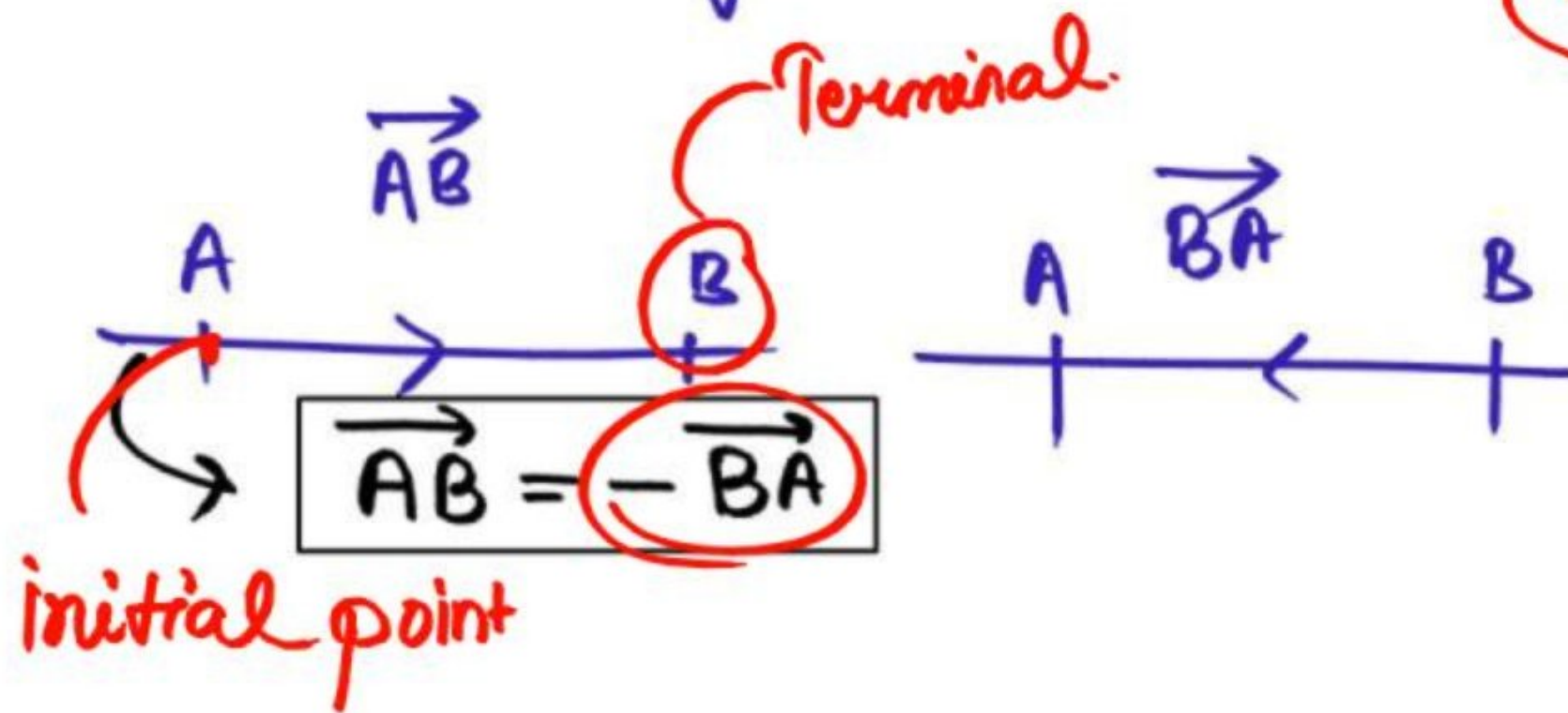
Vectors can be displace with ^{out} changed
its Magnitude and direction

Null Vector

Null vectors $\rightarrow$ Magnitude will be Zero

$$\text{Mag} = 0$$

Negative Vector



Equal Vector

Equal Magnitude and Same direction

Unit vector ($\hat{a}$)

$\vec{AB}$



$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\vec{a} = 2\hat{i} + 4\hat{j} + \hat{k}$$

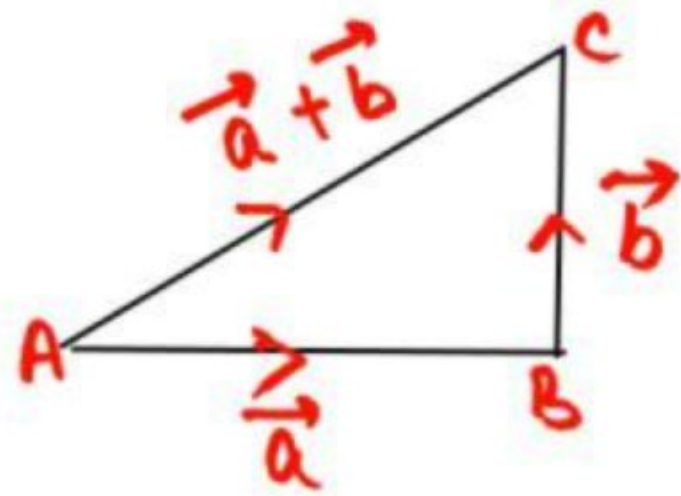
$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\hat{a} = \frac{2\hat{i} + 4\hat{j} + \hat{k}}{\sqrt{4 + 16 + 1}}$$

$$\hat{a} = \frac{2\hat{i} + 4\hat{j} + \hat{k}}{\sqrt{21}}$$

Triangle Law.

$$\vec{AB} + \vec{BC} = \vec{AC}$$



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$$\vec{a} = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$$

$$\vec{b} = x_2 \hat{i} + y_2 \hat{j} + z_2 \hat{k}$$

$$\vec{a} + \vec{b} = (x_1 + x_2) \hat{i} + (y_1 + y_2) \hat{j} + (z_1 + z_2) \hat{k}$$

$$\vec{a} - \vec{b} = (x_1 - x_2) \hat{i} + (y_1 - y_2) \hat{j} + (z_1 - z_2) \hat{k}$$

Position:-

Component Form of Position Vector

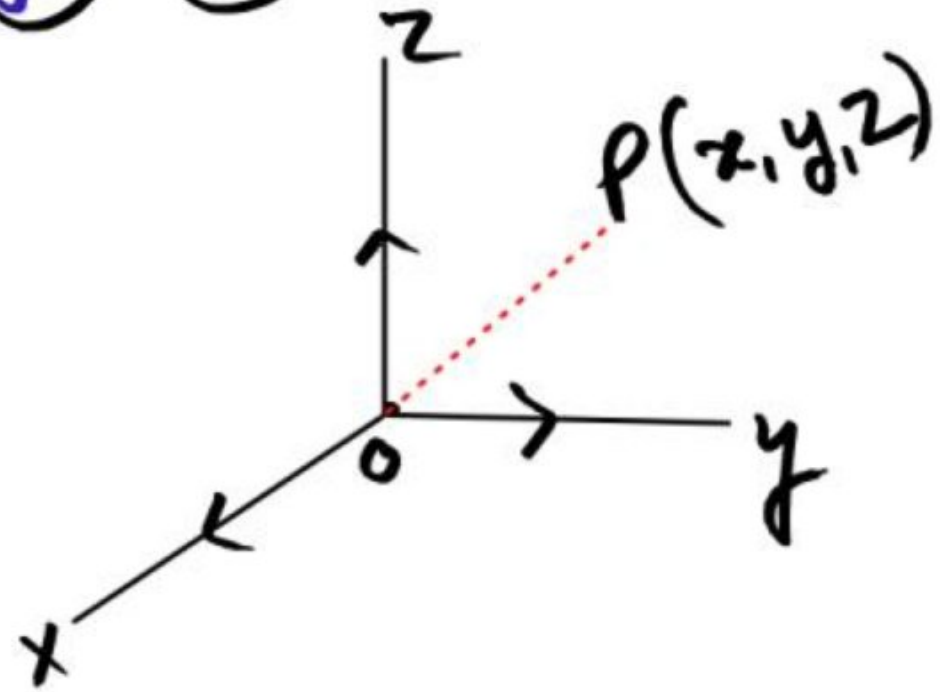
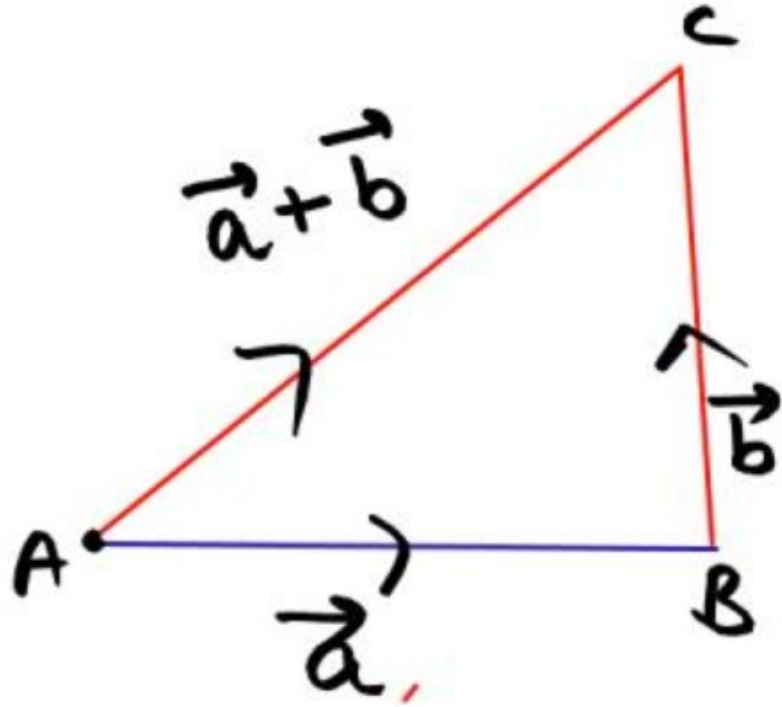
$$\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

x, y, z are the Scalar Component of $\vec{OP}$ $(x\hat{i}, y\hat{j}, z\hat{k})$ are vector component

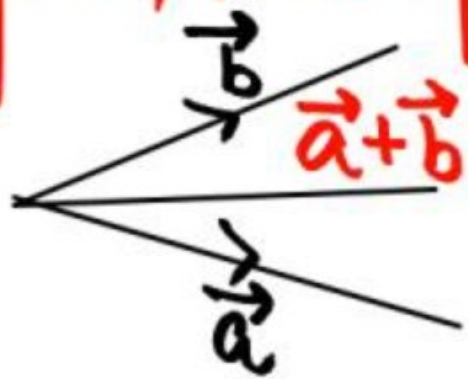
Triangle law of Add.

$$\vec{AB} + \vec{BC} = \vec{AC}$$

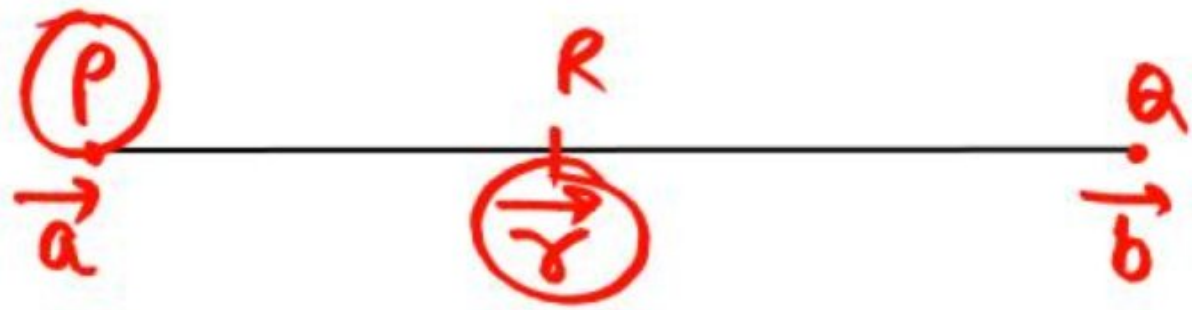
$$\vec{AC} = \vec{AC}$$



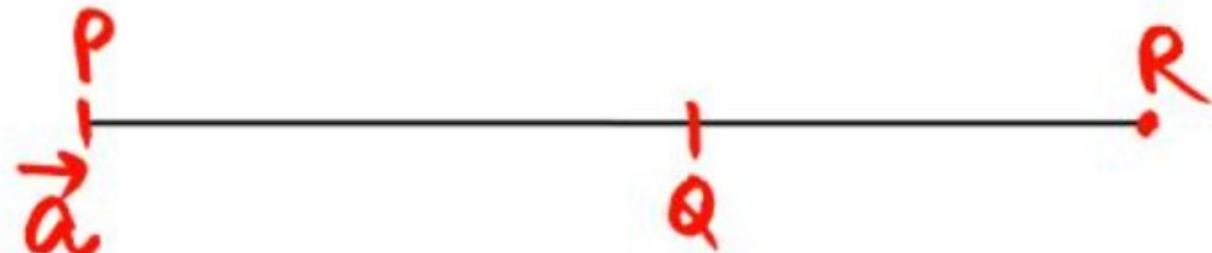
Parallelogram Law Addition



Section formula.



$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$$



$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$$

$$\frac{PQ}{QR} = \frac{m}{n}$$

Direction Cosine of a vectors

Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.

$$2\hat{i} + 3\hat{j} = x\hat{i} + y\hat{j}$$

$$x = 2$$

$$y = 3$$

ii) If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vector then write the value of $x + y + z$.

$$\vec{a} = \vec{b}$$

$$x\hat{i} + 2\hat{j} - z\hat{k} = 3\hat{i} - y\hat{j} + \hat{k}$$

$$x = 3 \quad y = -2 \quad z = -1$$

$$\begin{aligned} x + y + z &= 3 + (-2) + (-1) \\ &= 3 - 2 - 1 = 0 \text{ Ans} \end{aligned}$$

If $\vec{a} = x\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + z\hat{k}$, then find the value of x , y and z so that $2\vec{a} = 3\vec{b}$

$$\boxed{2\vec{a} = 3\vec{b}}$$

$$\vec{a} = x\hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{b} = 3\hat{i} - y\hat{j} + z\hat{k}$$

$$2[x\hat{i} + 2\hat{j} - 3\hat{k}] = 3[3\hat{i} - y\hat{j} + z\hat{k}]$$

$$2x\hat{i} + 4\hat{j} - 6\hat{k} = 9\hat{i} - 3y\hat{j} + 3z\hat{k}$$

$$\textcircled{x = \frac{9}{2}}$$

$$2x = 9$$

$$3z = -6$$

$$\boxed{z = -2}$$

$$-3y = 4$$

$$\textcircled{y = -\frac{4}{3}}$$

Find the magnitude of the following vectors:

i) $\hat{i} + \hat{j} + \hat{k}$

ii) $2\hat{i} - 3\hat{j} - 7\hat{k}$

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$$|\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2}$$

$$|\vec{a}| = \sqrt{3}$$

$$\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$$

ii) $2\hat{i} - 3\hat{j} - 7\hat{k}$

Let $\vec{b} = 2\hat{i} - 3\hat{j} - 7\hat{k}$

$$|\vec{b}| = \sqrt{4 + 9 + 49} = \sqrt{13 + 49} = \sqrt{62}$$

Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

$$\vec{a} + \vec{b} + \vec{c} = \hat{i} - 2\hat{j} + \hat{k} + (-2\hat{i} + 4\hat{j} + 5\hat{k}) + (\hat{i} - 6\hat{j} - 7\hat{k})$$

$$= \underline{\hat{i}} - 2\hat{j} + \hat{k} - \underline{2\hat{i}} + 4\hat{j} + 5\hat{k} + \underline{\hat{i}} - 6\hat{j} - 7\hat{k}$$

$$= \hat{i} - 2\hat{i} + \hat{i} - 2\hat{j} + 4\hat{j} - 6\hat{j} + \hat{k} + 5\hat{k} - 7\hat{k}$$

$$\boxed{\vec{a} + \vec{b} + \vec{c} = 0\hat{i} - 4\hat{j} - \hat{k}}$$

Find a unit vector in the direction of the vector $\vec{a} = 2\hat{i} - 3\hat{j}$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\vec{a} = 2\hat{i} - 3\hat{j}$$

$$|\vec{a}| = \sqrt{4 + 9} = \sqrt{13}$$

$$\hat{a} = \frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$$

Ans

Write a Unit vector in the direction of sum of vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $2\hat{i} + \hat{j} - 7\hat{k}$

If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$ find the unit vector in the direction of $2\vec{a} - \vec{b}$

$$2\vec{a} - \vec{b} = 2[\hat{i} + \hat{j} + 2\hat{k}] - [2\hat{i} + \hat{j} - 2\hat{k}]$$

$$= 2\hat{i} + 2\hat{j} + 4\hat{k} - 2\hat{i} - \hat{j} + 2\hat{k}$$

$$= 2\hat{i} - 2\hat{j} + 2\hat{j} - \hat{j} + 4\hat{k} + 2\hat{k}$$

$$2\vec{a} - \vec{b} = 0\hat{i} + \hat{j} + 6\hat{k}$$

$$\text{Unit Vector} \Rightarrow \hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$= \frac{\hat{j} + 6\hat{k}}{\sqrt{1+36}} = \frac{\hat{j} + 6\hat{k}}{\sqrt{37}}$$

Find a unit vector parallel to the sum of vectors $\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

i) Find a vector in the direction of the vectors $\vec{a} = \hat{i} - 2\hat{j}$, whose magnitude is 7 units.

Write a vector of magnitude 9 units in the direction of the vector $-2\hat{i} + \hat{j} + 2\hat{k}$.

Find Vectors of magnitude 5 unit which are parallel to the sum of vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

Write the value of p for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors.